

THERMOELASTIC MODELLING OF RECTANGULAR PLATE UNDER THE HYPERBOLIC HEAT CONDUCTION WITH AN INTERNAL HEAT SOURCE

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ABSTRACT

The present work is concerned with the study of thermal stress analysis of a two-dimensional rectangular plate under the hyperbolic heat conduction model with a heat source inside the body under the constant surface temperature. The internal heat source varying periodically with time is considered. The thermoelastic displacement, stresses and temperature are determined within the context of non-Fourier heat conduction theory. The solution for temperature field and stress field are obtained with the help of differential transform method and later used to find displacement. The effect of periodically varying heat source on thermal stresses is studied. The special case is considered and results are illustrated numerically and graphically for a copper plate.

KEYWORDS: Hyperbolic Heat Conduction Equation, Thermal Stress, Internal Heat Source, Differential Transform Method & Displacement Function

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INTRODUCTION

The study of the hyperbolic heat conduction equation in the field of thermoelasticity plays a vital role in many branches of science and technology. Thermoelasticity is the study of deformation occurs in the body due to thermal and mechanical load applied on it. As the advancement of technology, in many experimental results, it is observed that, heat is conducted in a material with a finite speed which contradicts the fact of infinite heat of propagation which is explained well by Fourier law of heat conduction or parabolic heat conduction. The heat is transported in hyperbolic form when the temperature gradient is very high, extremely large heat fluxes and time duration is very short. Cattaneo [1] and Vernotte [2] proposed a modification in the classical theory of heat conduction by bringing a new term called relaxation time for heat flux in solids, which is given by the relation

$$\mathbf{q} + \tau_q \frac{\partial \mathbf{q}}{\partial t} = -\lambda \nabla T \quad (1)$$

where, τ_q is called relaxation time, $\mathbf{q}$ is the heat flux and λ is the thermal conductivity of the material.

Most of the research works on the thermal response of rectangular plate in the parabolic heat conduction domain are available in the literature. V. R. Manthane et.al [3] obtained the solution of the transient thermoelastic problem of non-homogenous rectangular plate using integral transform technique. N. Sumi [4] investigated the thermal stress in a finite rectangular plate by the complex method. T. C. Lim [5] investigated the thermal stress in a thin auxetic plate and provides a different choice to decrease the thermal stress. Chen [6] has studied the thermal stress in a rectangular plate subjected to non- uniform heat transfer in terms of power series using Lanczos-

Chebyshev and discrete least square methods. Tanigawa and Komatsubara [7] have discussed the thermal stress and its intensity factor under compressive stress field in a rectangular plate. From the last few decades, the researcher works on non-Fourier heat conduction. As per El Dhaba and Abou-Dina [8] analysis, the thermal stress in a long cylinder of rectangular cross section occurs due to variable heat sources and variable pressure at its boundary in finite Fourier transforms domain. Chen and Hu [9] suggested that HHC model are more efficient to design structure against fracture under thermal loading. Wang and Li [10] studied the HHCE for the fracture of a finite piezoelectric layer using Laplace and dual integral transform and also investigated thermal shock resistance in a plate under HHC model. Ahmed [11] calculated the thermal stresses numerically for a nano composite slab. M. R. Talaei and V. Sarafrazi [12] presented an analytical solution for non Fourier heat conduction in a solid of rectangular shape using Eigen function expansion and shows the use of laser absorption in tissues. H. R. Al-Duhaim et.al [13] determined the temperature distribution and thermal stresses for the laser short pulse heating of solid surface considering HHC model.

Yang [14] has applied a sequential method to find the solution of direct and inverse HHCE using finite difference and the Newton Raphson method. N. S. Al-Huniti and M. A. Al-N.M. Naji [15] worked on the thermoelastic nature of rod under the effect of moving heat source considering HHC model. L. Ping et al. [16] worked on the higher-order thermoelastic response to determine the thermal bending of the laminated composite plate. M. H. Hsu [17] presented the solution of the hyperbolic heat conduction equation and also investigated the impact of relaxation time on temperature distribution. I.V. kudinov and V.A. kudinov [18] studied the dynamic stresses in an infinite plate associated with HHCE. B.A. Hamid [19] worked on an analytical solution of non-Fourier heat conduction equation with periodic thermal oscillation using integral transform technique. In past literature, many authors used DTM (Differential Transform Method) to solve different initial and boundary value problems and showed the result obtained is much closer to the exact solution. Afshin Babaei et.al [20] discussed the inverse problem of heat conduction using reduced DTM. Shaher momani and Zaid odibat [21] discussed the solution of non-linear fractional partial differential equation applying DTM. Jafari, Sadeghi and Biswas [22] have studied an analytical solution of multi-dimensional partial differential equation by DTM. This method, due to its simplicity and effectiveness are used by many researchers.

The present work is concerned with the study of thermal stress analysis of a two-dimensional rectangular plate under the hyperbolic heat conduction model with a periodically varying heat source inside the body under the constant surface temperature. The thermoelastic displacement, stresses and temperature are determined within the context of the generalized theory of thermoelasticity. The solution for temperature field and stress field are obtained with the help of differential transform method. The effect of periodically varying heat source on induced thermal stresses in a plate is illustrated numerically and graphically for a copper plate.

MATHEMATICAL MODELLING FOR TEMPERATURE FIELD

Consider a plate of rectangular shape occupying the space $D: \{(x, y, t): x \in [0, a], y \in [0, b], t \in [0, \tau]\}$. The unsteady state temperature distribution under the hyperbolic heat conduction in the rectangular plate with heat generation inside the body is given by

$$\frac{\partial T}{\partial t} + \tau \frac{\partial^2 T}{\partial t^2} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{Q}{\rho c} \quad (2)$$

where, $k = \frac{\lambda}{\rho c}$, ρ , c are the thermal diffusivity, mass density and specific heat capacity respectively and Q is the internal

heat source.

With initial and boundary conditions are given by

$$T(x, y, 0) = T_0 \quad (3)$$

$$\frac{\partial T(x, y, 0)}{\partial \tau} = 0 \quad (4)$$

$$T(0, y, t) = T_s \quad (5)$$

$$T(x, a, t) = 0 \quad (6)$$

$$T(x, y, t) = 0 \quad (7)$$

$$T(x, b, t) = 0 \quad (8)$$

Solution for Temperature Field

Considering $T_0 = \sum_{m=n=1}^{\infty} M_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$ and $T_s = \sum_{m=n=1}^{\infty} M_{mn} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)$

And solving of the problem defined from eq.(2) to eq.(8) is done semi analytically using differential transform method given in reference [16]. Thus, after applying DTM on eq.(2) to eq.(8), one will get

$$T(\zeta, \eta, \Theta + 2) = \frac{1}{\tau_q(\Theta+1)(\Theta+3)} \left[\kappa(\zeta+1)(\zeta+2)T(\zeta+2, \eta, \Theta) + \kappa(\eta+1)(\eta+2)T(\zeta, \eta+2, \Theta) - (\zeta+1)T(\zeta, \eta, \Theta+1) + \frac{Q(\zeta, \eta, \Theta)}{\rho c} \right]$$

$$T(\zeta, \eta, 0) = \sum_{m=n=1,3,5,\dots}^{\infty} \frac{16T_0}{mn\pi^2} \frac{m^{\zeta} n^{\eta} \pi^{\zeta+\eta} (-1)^{\zeta+\eta-2/2}}{\zeta! \eta!}, T(\zeta, \eta, 1) = 0 \quad (9)$$

$$T(0, \eta, \Theta) = \sum_{m=1,3,5,\dots}^{\infty} \frac{16T_0}{mn\pi^2} \frac{m^{\Theta} n^{\eta} \pi^{\Theta+\eta} (-1)^{\Theta+\eta-2/2}}{\Theta! \eta!}, T(\zeta, 0, \Theta) = 0 \quad (10)$$

Now, using these transformed boundary and initial conditions in eq.(9), one can obtain $T(\zeta, \eta, \Theta)$ for different values of ζ, η, Θ and which is given as:

$$T(\zeta, \eta, \Theta) = \sum_{m=n=1,3,5,\dots}^{\infty} \frac{16T_0}{mn\pi^2} T(\zeta, \eta, 0) \sum_{p=\Theta/2}^1 \frac{A(-1)^p k^p}{\Theta! \tau_q^{\Theta}} (m^2 \pi^2 + n^2 \pi^2)^p + \frac{Q(\zeta, \eta, \Theta)}{\rho c} \quad (11)$$

where, ζ, η are odd and Θ is even

$$T(\zeta, \eta, \Theta) = \sum_{m=n=1,3,5,\dots}^{\infty} \frac{16T_0}{mn\pi^2} T(\zeta, \eta, 0) \sum_{p=\frac{\Theta-1}{2}}^1 \frac{B(-1)^{p-1} k^p \tau_q^p}{(\Theta-1)! \tau_q^{\Theta}} (m^2 \pi^2 + n^2 \pi^2)^p + \frac{Q(\zeta, \eta, \Theta)}{\rho c} \quad (12)$$

where, ζ, η, Θ are odd

Now, we considered here the heat source in rectangular co-ordinates $Q(x, y, t)$ which varies periodically in the following form:

$$Q(x, y, t) = Q_0 \frac{\delta(x) \delta(y)}{ab} \sin\left(\frac{\pi t}{\tau}\right), \quad 0 \leq t \leq \tau$$

$$= 0 \quad t > \tau \quad (13)$$

Where Q_0 , the strength of the heat source and δ is the Dirac's delta function.

On applying differential transform on equation (13), we get

$$Q(\zeta, \eta, \theta) = Q_0 \frac{\delta(\zeta) \delta(\eta)}{ab} \frac{(\tau/\tau)^{\theta}}{\theta!} \sin\left(\frac{\pi \theta}{\tau}\right) \quad (14)$$

By applying inverse differential transform and using equation (11), (12) and (14), one can obtain series solution for $T(x, y, t)$ which is as follows:

$$T(x, y, t) = \sum_{\zeta=0}^{\infty} \sum_{\eta=0}^{\infty} \sum_{\theta=0}^{\infty} T(\zeta, \eta, \theta) x^{\zeta} y^{\eta} t^{\theta} \quad (15)$$

Using $T(\zeta, \eta, \theta)$ for each values of ζ, η, θ from eq. (11), (12) and using (14) in eq. (15), we can get the solution for temperature function as:

$$T(x, y, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} \frac{16V_0}{mn\pi^2} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) + \frac{Q_0}{\tau a b \tau} \left(\frac{\tau}{\tau}\right)^{\theta} \left(\frac{\pi \tau}{\tau} - \sin\frac{\pi \tau}{\tau}\right) - \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} \frac{16V_0}{mn\pi^2} \sin\left(\frac{m\pi x}{a}\right) \left(\frac{m\pi}{\tau} - \sin\frac{m\pi \tau}{\tau}\right) +$$

$$k \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{p=0}^{\infty} \frac{16V_0}{mn\pi^2} \sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) (m^2 \pi^2 + n^2 \pi^2) \left\{ \sum_{\theta=2}^{\infty} \frac{(-1)^{\theta} (\tau/\tau)^{\theta}}{\theta!} + k(m^2 \pi^2 + n^2 \pi^2) \tau a^2 \sum_{\theta=2}^{\infty} \frac{(-1)^{\theta} (\theta-1) \tau^{\theta+2}}{\tau a^2 \theta! (\theta+2)!} \right\} \quad (16)$$

MATHEMATICAL MODELLING FOR THERMAL STRESSES

Considering thermal stress function χ in rectangular coordinates system defined in reference [23], as given by

$$\chi = \chi_c + \chi_p.$$

Where χ_c is a complementary function and χ_p is a particular integral.

And χ_c and χ_p are satisfied by the equations

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \chi_c = 0 \quad (17)$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) \chi_p = -\alpha E T \quad (18)$$

And component of stress function are given by

$$\sigma_{xx} = \frac{\partial^2 \chi}{\partial y^2} \quad (19)$$

$$\sigma_{yy} = \frac{\partial^2 \chi}{\partial x^2} \quad (20)$$

$$\sigma_{xy} = -\frac{\partial^2 \chi}{\partial x \partial y} \quad (21)$$

And the boundary conditions

$$\sigma_{xx} = 0, \sigma_{xy} = 0 \quad \text{on } x = 0, a \quad (22)$$

Solution for Stress Function

The complementary function χ_c can be obtained by solving eq.(17).

$$\text{Let } \chi_c = \frac{c_1}{2} x^2 y + \frac{c_2}{2} x y^2 + \frac{c_3}{6} y^3 \quad (23)$$

Solution of particular integral χ_p is obtained by applying differential transform on eq. (18). The transformed form of eq.(18) is given by

$$(\mu + 2)(\mu + 1)V(\mu + 2, \nu, \omega) + (\nu + 2)(\nu + 1)V(\mu, \nu + 2, \omega) = -\alpha E \sum_{\zeta=0}^{\infty} \sum_{\eta=0}^{\infty} \sum_{\theta=0}^{\infty} T(\zeta, \eta, \theta) \delta(\mu - \zeta, \nu - \eta, \omega - \theta) \quad (24)$$

where, $V(\mu, \nu, \omega)$ is the transformed function of χ_p .

For each μ, ν, ω the values $V(\mu, \nu, \omega)$ by recursive method can be evaluated as:

$$V(\mu, \nu, \omega) = \sum_{\zeta=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta!} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta - 2 - 2r)! \frac{(\eta+2r)!}{\eta!} T(\zeta - 2 - 2r, \eta + 2r, \theta) \right) + \sum_{\zeta=2, \eta=0, \theta=1}^{\infty} \sum_{l=1}^{\infty} \frac{Q_0}{\tau_q \rho c} \left(\frac{1}{\pi} \right)^2 \frac{(\pi/\tau)^l}{l!} \delta(\zeta - 2, \eta, \theta - l) \quad (25)$$

Now applying the definition of inverse differential transform, one can obtain χ_p as:

$$\chi_p = -\alpha E \sum_{\zeta=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta!} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta - 2 - 2r)! \frac{(\eta+2r)!}{\eta!} T(\zeta - 2 - 2r, \eta + 2r, \theta) \right) x^{\zeta} y^{\eta} t^{\theta} + \frac{\alpha E}{2!} \frac{Q_0}{\tau_q \rho c} \left(\frac{1}{\pi} \right)^2 \left(\frac{\pi t}{\tau} - \sin \frac{\pi t}{\tau} \right) x^2 \quad (26)$$

Using eq. (23) and eq. (25), we will get expression for stress function χ , which is given by

$$\chi = \frac{c_1}{2} x^2 y + \frac{c_2}{2} x y^2 + \frac{c_3}{6} y^3 - \alpha E \sum_{\zeta=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta!} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta - 2 - 2r)! \frac{(\eta+2r)!}{\eta!} T(\zeta - 2 - 2r, \eta + 2r, \theta) \right) x^{\zeta} y^{\eta} t^{\theta} + \frac{\alpha E}{2!} \frac{Q_0}{\tau_q \rho c} \left(\frac{1}{\pi} \right)^2 \left(\frac{\pi t}{\tau} - \sin \frac{\pi t}{\tau} \right) x^2 \quad (27)$$

And now using this value of stress function in eq. (19)~(21), we will get stress component as:

$$\sigma_{xx} = c_2 x + c_3 y - \alpha E \sum_{\zeta=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta!} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta - 2 - 2r)! \frac{(\eta+2r)!}{\eta!} T(\zeta - 2 - 2r, \eta + 2r, \theta) \right) x^{\zeta} y^{\eta} t^{\theta} \quad (28)$$

$$\sigma_{yy} = c_1 y - \alpha E \sum_{\zeta=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta!} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta - 2 - 2r)! \frac{(\eta+2r)!}{\eta!} T(\zeta - 2 - 2r, \eta + 2r, \theta) \right) x^{\zeta-2} y^{\eta} t^{\theta} + \frac{\alpha E}{2!} \frac{Q_0}{\tau_q \rho c} \left(\frac{1}{\pi} \right)^2 \left(\frac{\pi t}{\tau} - \sin \frac{\pi t}{\tau} \right) \quad (29)$$

$$\sigma_{xy} = - \left[c_1 x + c_2 y - \alpha E \sum_{\zeta=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta-1!} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! \frac{(\eta+2r)!}{\eta-1!} T(\zeta-2-2r, \eta+2r, \theta) \right) x^{\zeta-1} y^{\eta-1} t^{\theta} + \frac{\alpha E}{1!} \frac{Q_0}{\tau_0 \rho c} \left(\frac{\tau}{\pi} \right)^2 \left(\frac{\pi \tau}{\tau} - \sin \frac{\pi \tau}{\tau} \right) x \right] \quad (30)$$

Now applying the boundary condition defined in eq. (22), we will get

$$c_2 = c_3 = 0$$

$c_1 = \alpha E \sum_{\zeta=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta!} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! (1+2r)! T(\zeta-2-2r, 1+2r, \theta) \right) \alpha^{\zeta-1} t^{\theta} - \frac{\alpha E}{1!} \frac{Q_0}{\tau_0 \rho c} \left(\frac{\tau}{\pi} \right)^2 \left(\frac{\pi \tau}{\tau} - \sin \frac{\pi \tau}{\tau} \right)$ Now substituting these values of c_1, c_2, c_3 in eq. (28)~(30), one will get the expression for stress components as:

$$\sigma_{xx} = -\alpha E \sum_{\zeta=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta!} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! \frac{(\eta+2r)!}{\eta-1!} T(\zeta-2-2r, \eta+2r, \theta) \right) x^{\zeta} y^{\eta-2} t^{\theta} \quad (31)$$

$$\sigma_{yy} = \alpha E \sum_{\zeta=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta!} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! (1+2r)! T(\zeta-2-2r, 1+2r, \theta) \right) \alpha^{\zeta-1} t^{\theta} - \frac{\alpha E}{1!} \frac{Q_0}{\tau_0 \rho c} \left(\frac{\tau}{\pi} \right)^2 \left(\frac{\pi \tau}{\tau} - \sin \frac{\pi \tau}{\tau} \right) \alpha y + \alpha E \sum_{\zeta=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta-1!} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! \frac{(\eta+2r)!}{\eta-1!} T(\zeta-2-2r, \eta+2r, \theta) \right) x^{\zeta-2} y^{\eta} t^{\theta} + \frac{\alpha E}{1!} \frac{Q_0}{\tau_0 \rho c} \left(\frac{\tau}{\pi} \right)^2 \left(\frac{\pi \tau}{\tau} - \sin \frac{\pi \tau}{\tau} \right) \quad (32)$$

$$\sigma_{xy} = -\alpha E \sum_{\zeta=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta!} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! (1+2r)! T(\zeta-2-2r, 1+2r, \theta) \right) \alpha^{\zeta-1} t^{\theta} - \frac{\alpha E}{1!} \frac{Q_0}{\tau_0 \rho c} \left(\frac{\tau}{\pi} \right)^2 \left(\frac{\pi \tau}{\tau} - \sin \frac{\pi \tau}{\tau} \right) \alpha x + \alpha E \sum_{\zeta=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta-1!} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! \frac{(\eta+2r)!}{\eta-1!} T(\zeta-2-2r, \eta+2r, \theta) \right) x^{\zeta-1} y^{\eta-1} t^{\theta} - \frac{\alpha E}{1!} \frac{Q_0}{\tau_0 \rho c} \left(\frac{\tau}{\pi} \right)^2 \left(\frac{\pi \tau}{\tau} - \sin \frac{\pi \tau}{\tau} \right) x \quad (33)$$

MATHEMATICAL MODELLING FOR DISPLACEMENT FUNCTION

The displacement function defined in reference [23] for plane stress in a rectangular coordinate is given by

$$U_x = \frac{1}{2G} \left[-\frac{\partial \chi}{\partial x} + \frac{1}{1+\nu} \frac{\partial \psi}{\partial y} \right] \quad (34)$$

$$U_y = \frac{1}{2G} \left[-\frac{\partial \chi}{\partial y} + \frac{1}{1+\nu} \frac{\partial \psi}{\partial x} \right] \quad (35)$$

where, G and ν are shear modulus of elasticity and poisson's ratio, and χ, ψ satisfy the following equation as

$$\sigma_{xx} + \sigma_{yy} + \alpha E T \equiv \frac{\partial^2 \psi}{\partial x \partial y} \text{ and } \frac{\partial^2}{\partial x \partial y} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \psi = 0 \quad (36)$$

Solution for Displacement Function

To calculate displacement function given by eq. (34) & (35), firstly, one must calculate $\frac{\partial \psi}{\partial x}$ & $\frac{\partial \psi}{\partial y}$ which can be determined by taking integration on eq.(36) with respect to x & y respectively, therefore

$$\begin{aligned} \frac{\partial \psi}{\partial x} = & -\alpha E \sum_{r=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta-1} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! \frac{(\eta+2r)!}{\eta!} T(\zeta-2-2r, \eta+2r, \theta) \right) \left[\frac{x^2}{(\zeta-1)(\eta-1)!} + \frac{y^2}{(\zeta-2)(\eta+1)!} \right] x^{\zeta-2} y^{\eta-1} t^{\theta} + \\ & \alpha E \sum_{r=2}^{\infty} \sum_{\theta=1}^{\infty} \sum_{\eta=1}^{\infty} \frac{1}{\zeta-1} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! (1+2r)! T(\zeta-2-2r, 1+2r, \theta) \right) \alpha^{\zeta-1} t^{\theta} \frac{y^2}{2} + \frac{\alpha E}{1!} \frac{Q_0}{\tau_0 \rho c} \left(\frac{\tau}{\pi} \right)^2 \left(\frac{\pi t}{\tau} - \right. \\ & \left. \sin \frac{\pi t}{\tau} \right) \left(\alpha \frac{y^2}{2} - y \right) + \alpha E \left(\sum_{r=1}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=2}^{\infty} T(\zeta, \eta, \theta) \frac{x^{\zeta} y^{\eta+1}}{\eta+1} t^{\theta} + \sum_{\eta=1}^{\infty} \sum_{\theta=2}^{\infty} T(0, \eta, \theta) \frac{y^{\eta+1}}{\eta+1} t^{\theta} \right) + \phi(x) \end{aligned} \quad (37)$$

$$\begin{aligned} \frac{\partial \psi}{\partial y} = & -\alpha E \sum_{r=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta-1} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! \frac{(\eta+2r)!}{\eta!} T(\zeta-2-2r, \eta+2r, \theta) \right) \left[\frac{x^2}{(\zeta+1)(\eta-2)!} + \frac{y^2}{(\zeta-1)(\eta)!} \right] x^{\zeta-1} y^{\eta-2} t^{\theta} + \\ & \alpha E \sum_{r=2}^{\infty} \sum_{\theta=1}^{\infty} \sum_{\eta=1}^{\infty} \frac{1}{\zeta-1} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! (1+2r)! T(\zeta-2-2r, 1+2r, \theta) \right) \alpha^{\zeta-1} y x t^{\theta} - \frac{\alpha E}{1!} \frac{Q_0}{\tau_0 \rho c} \left(\frac{\tau}{\pi} \right)^2 \left(\frac{\pi t}{\tau} - \right. \\ & \left. \sin \frac{\pi t}{\tau} \right) (\alpha x y - x) + \alpha E \left(\sum_{r=1}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=2}^{\infty} T(\zeta, \eta, \theta) \frac{x^{\zeta+1} y^{\eta}}{\zeta+1} t^{\theta} + \sum_{\eta=1}^{\infty} \sum_{\theta=2}^{\infty} T(0, \eta, \theta) x y^{\eta} t^{\theta} \right) + \phi(y) \end{aligned} \quad (38)$$

Using the above values in eq. (34) & (35) and neglecting the rigid deformation, one can get the displacement function as:

$$\begin{aligned} U_x = & \frac{-\alpha E}{2\theta} \left[-\sum_{r=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta-1} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! \frac{(\eta+2r)!}{\eta!} T(\zeta-2-2r, \eta+2r, \theta) \right) \left[\left(1 + \frac{1}{1+\nu} \right) \frac{x^2}{(\zeta)(\eta-1)!} + \right. \right. \\ & \left. \left. \frac{y^2}{(\zeta-2)(\eta+1)!} \right] x^{\zeta-2} y^{\eta-1} t^{\theta} + \sum_{r=2}^{\infty} \sum_{\theta=1}^{\infty} \sum_{\eta=1}^{\infty} \frac{1}{\zeta-1} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! (1+2r)! T(\zeta-2-2r, 1+2r, \theta) \right) \alpha^{\zeta-1} t^{\theta} \left[\frac{y^2}{2} - \right. \right. \\ & \left. \left. \frac{1}{1+\nu} \frac{y^2}{2} \right] - \frac{Q_0}{\tau_0 \rho c} \left(\frac{\tau}{\pi} \right)^2 \left(\frac{\pi t}{\tau} - \sin \frac{\pi t}{\tau} \right) \left[\frac{\alpha x^2}{2} - \frac{\alpha}{1+\nu} \frac{y^2}{2} + \frac{y}{1+\nu} \right] - \right. \\ & \left. \left(\sum_{r=1}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=2}^{\infty} T(\zeta, \eta, \theta) \frac{x^{\zeta} y^{\eta+1}}{\eta+1} \frac{t^{\theta}}{1+\nu} + \sum_{\eta=1}^{\infty} \sum_{\theta=2}^{\infty} T(0, \eta, \theta) \frac{y^{\eta+1}}{\eta+1} \frac{t^{\theta}}{1+\nu} \right) \right] \end{aligned} \quad (39)$$

$$\begin{aligned} U_y = & \frac{-\alpha E}{2\theta} \left[\sum_{r=2}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=1}^{\infty} \frac{1}{\zeta-1} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! \frac{(\eta+2r)!}{\eta!} T(\zeta-2-2r, \eta+2r, \theta) \right) \left[\left(\frac{1}{1+\nu} \right) \left(\frac{x^2}{(\zeta)(\eta-2)!} + \frac{y^2}{(\zeta-2)(\eta)!} \right) - \right. \right. \\ & \left. \left. \frac{\alpha y^2}{(\zeta-1)(\eta+1)!} \right] x^{\zeta-2} y^{\eta-2} t^{\theta} + \sum_{r=2}^{\infty} \sum_{\theta=1}^{\infty} \sum_{\eta=1}^{\infty} \frac{1}{\zeta-1} \sum_{r=0}^{\zeta-2} \left((-1)^r (\zeta-2-2r)! (1+2r)! T(\zeta-2-2r, 1+2r, \theta) \right) \alpha^{\zeta-1} t^{\theta} y \left[x - \frac{1}{1+\nu} \right] + \right. \\ & \left. \frac{Q_0}{\tau_0 \rho c} \left(\frac{\tau}{\pi} \right)^2 \left(\frac{\pi t}{\tau} - \sin \frac{\pi t}{\tau} \right) (\alpha y - 1) \left[x - \frac{1}{1+\nu} \right] - \left(\sum_{r=1}^{\infty} \sum_{\eta=1}^{\infty} \sum_{\theta=2}^{\infty} T(\zeta, \eta, \theta) \frac{x^{\zeta+1} y^{\eta}}{\zeta+1} \frac{t^{\theta}}{1+\nu} + \sum_{\eta=1}^{\infty} \sum_{\theta=2}^{\infty} T(0, \eta, \theta) \frac{x y^{\eta}}{1} \frac{t^{\theta}}{1+\nu} \right) \right] \end{aligned} \quad (40)$$

Table 1: Properties of Copper

α	$17 \times 10^{-6} ({}^{\circ}\text{C})^{-1}$
τ_0	0.02s
ν	0.33
G	4.5×10^{10}
E	$1.19 \times 10^{11} \text{ N/m}^2$
k	$1.1283 \times 10^{-4} \text{ m}^2/\text{s}$

NUMERICAL RESULTS AND GRAPHICAL DISCUSSIONS

To obtain the numerical results, the copper plate is used for which material parameter are taken from the reference [24] which are given in table 1. The stress variation along spatial direction is determined and plotted. All the values are calculated taking $a = 1\text{cm}$, $b = 1\text{cm}$, $\tau = 1\text{s}$, $T_0 = 0$ and $T_s = 10^{\circ}\text{C}$

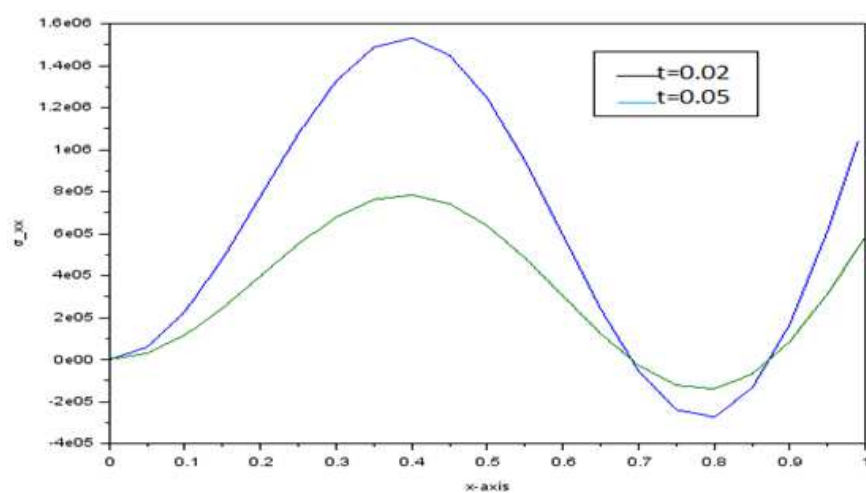


Figure 1: Stress Component XX Along X Direction.

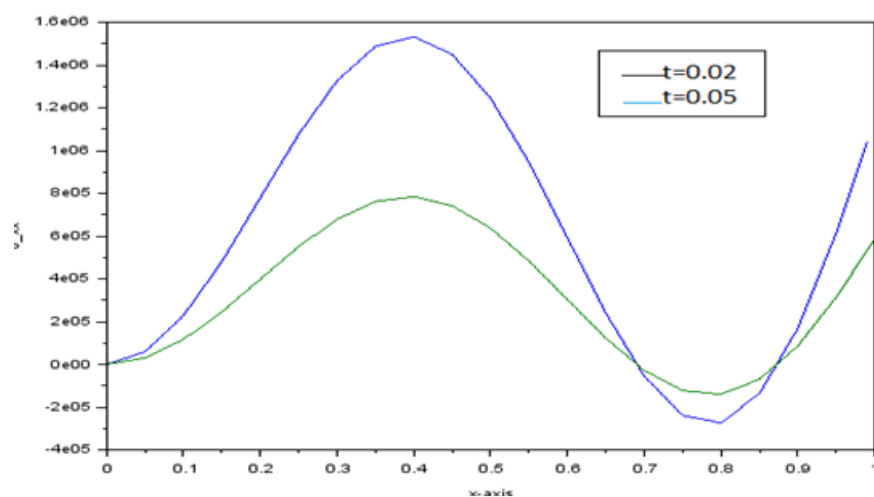


Figure 2: Stress Component YY Along Y Direction.

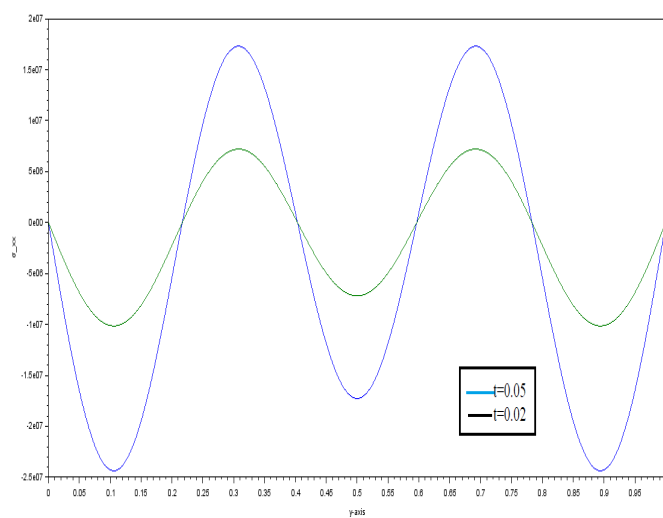


Figure 3: Stress Component XX Along Y Direction.

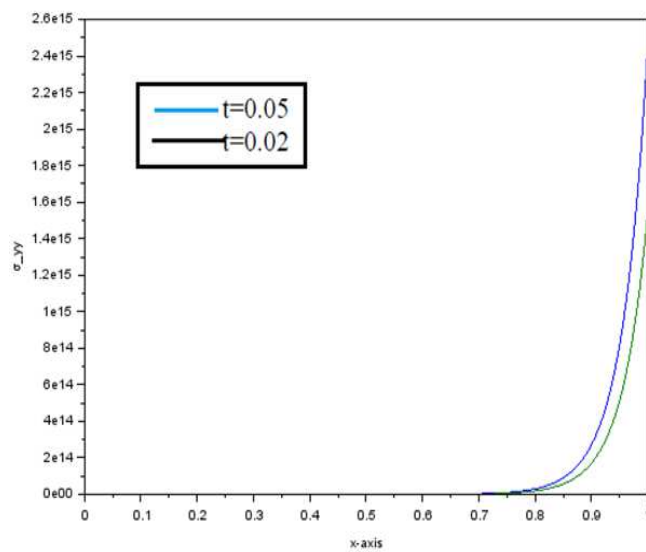


Figure 4: Stress Component YY Along X Direction

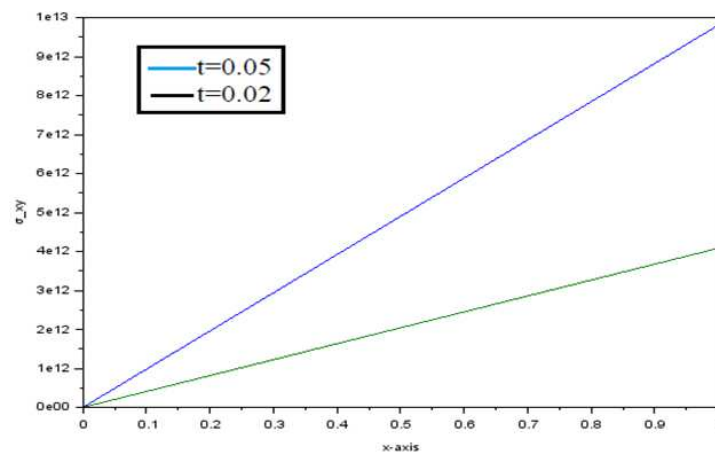


Figure 5: Stress Component XY Along X Direction.

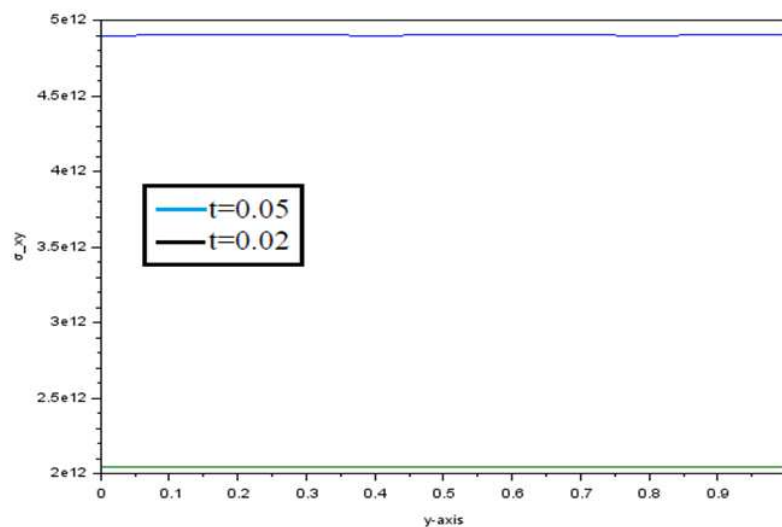


Figure 6: Stress Component XY Along Y Direction.

Figure 1 shows the variation of thermal stress component σ_{xx} , along x-direction. it seems that along x-axis compressive and tensile stress occurs due to the constant surface temperature. It is also observed from the figure that compressive stress occurs only between $0.7 \leq x \leq 0.9$ and in remaining part tensile stress is observed.

Figure 2 shows the variation of thermal stress component σ_{yy} , occurred in mid-point of the plate along y-direction. It seems that, initially no stress occurs in the plate and it decreases gradually as move along the outer edge of the plate as an effect of periodically varying internal heat source. It is also shown from the graph, as the time increases, the strength of stress increases in both the cases.

Figure 3 shows the variation of thermal stress component σ_{xy} , along the vertical direction. It is observed from the figure that the values of stress component oscillate from compressive to tensile throughout along vertical direction. It is also clear from the figure that the nature of the stress component σ_{xy} is periodical.

Figure 4 shows the variation of thermal stress component σ_{yy} , along horizontal direction. From the figure, it observed that initially there is no stress along x-direction but after the middle of the plate stress appears in a plate and continuously increases up to the end of the plate.

Figure 5 shows the variation of thermal stress component σ_{xy} , along the horizontal direction. From the figure, it is observed that σ_{xy} varies linearly and shows maximum value at the boundary of the plate.

Figure 6 shows the variation of thermal stress component σ_{xy} , along the vertical direction. It is observed from the figure that the stress is constant throughout the plate along y-direction.

CONCLUSIONS

In the present work, thermal stress analysis of rectangular plate under the hyperbolic heat conduction model has been investigated. The response of the heat conduction equation under periodically varying heat source is also studied. The distribution of temperature, stress and displacement are obtained under the differential transform domain. From the numerical result and discussion, it is observed that

- The stress component σ_{xx} is compressive and tensile in nature due to constant surface temperature and as an effect of relaxation time.
- The stress component σ_{yy} shows opposite nature along spatial direction due to the periodically varying internal heat source
- The stress component σ_{xy} shows different nature along x and y direction.
- The strength of all the stress component increases as increase in time.

From the present analysis, it is also observed that the periodically varying heat source affect the nature of the thermal stresses. And, also the internal heat source influences the temperature distribution under the hyperbolic heat conduction model. These results may be useful in the study of thermal characteristics of the various bodies in design and engineering applications.

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